Meeting: 1001, Evanston, Illinois, SS 8A, Special Session on Computability Theory and Applications

1001-03-98 **Stephen G. Simpson*** (simpson@math.psu.edu), Department of Mathematics, Pennsylvania State University, State College, PA 16802. *Mass Problems to the Rescue!*

Two unsolved problems concerning the semilattice $\mathcal{R}_T$ of r. e. Turing degrees are: (1) To find a specific, natural example of an r. e. degree other than $0'$ and 0. (2) To find a "smallness property" of an infinite co-r. e. set which ensures that its degree is $< 0'$. We now widen the context to obtain satisfactory solutions to both problems. Consider the lattice $\mathcal{P}_w$ of weak degrees of mass problems associated with nonempty Π^0_1 subsets of 2^ω. There is a natural embedding of $\mathcal{R}_T$ into $\mathcal{P}_w$ carrying $0'$ and 0 to the top and bottom degrees in $\mathcal{P}_w$ respectively. We identify $\mathcal{R}_T$ with its image under this embedding. Regarding (1), there are many natural degrees in $\mathcal{P}_w$ related to foundationally interesting topics such as reverse mathematics, algorithmic randomness, subrecursive hierarchies, and computational complexity. Unfortunately, these natural degrees in $\mathcal{P}_w$ are incomparable with all of the r. e. degrees, except $0'$ and 0. Regarding (2), there are smallness properties of nonempty Π^0_1 sets $P \subseteq 2^\omega$ analogous to hypersimplicity etc. which imply that the weak degree of P is $< 0'$. (Some of this work is joint with Stephen Binns.) (Received August 16, 2004)