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1001-03-44 **Rodney G. Downey** and **Carl G. Jockusch*** (jockusch@math.uiuc.edu), Department of Mathematics, University of Illinois, 1409 E Green St., Urbana, IL 61801, and **Joseph S. Miller**.
Degrees of nontrivial self-embeddings of computable linear orderings.

The Dushnik–Miller Theorem states that every infinite countable linear ordering has a nontrivial self-embedding. We study the degrees of nontrivial self-embeddings of computable linear orderings of ω. We show that there is a discrete computable linear ordering L of ω such that every computable nontrivial self-embedding of L has degree $\mathbf{a} >> \mathbf{0}'$. (Here $\mathbf{a} >> \mathbf{0}'$ means that every infinite $\mathbf{0}'$-computable tree of binary strings has an infinite $\mathbf{a}$-computable path.) In the other direction, we show that if L is a computable linear ordering of ω and the set of elements of L which have a successor is Δ^0_2, then L has an $\mathbf{a}$-computable self-embedding for every degree $\mathbf{a} >> \mathbf{0}'$. Finally, we use a $0'''$-argument to show that there is a computable linear ordering L of ω such that every block of L is finite and L has no Δ^0_2 nontrivial self-embedding.

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