Meeting: 999, Nashville, Tennessee, SS 7A, Special Session on Operator Theory on Function Spaces

999-47-225 **Keiji Izuchi*** (`izuchi@math.sc.niigata-u.ac.jp`), Department of Mathematics, Niigata University, 950-2181 Niigata, Japan. *Backward shift invariant subspaces in the bidisc.*

Let Γ^2 be the 2-dimensional unit torus. We denote by $(z, w) = (e^{i\theta}, e^{i\phi})$ the variables in $\Gamma^2 = \Gamma_z \times \Gamma_w$. For every invariant subspace M in the Hardy spaces $H^2(\Gamma^2)$, let R_z and R_w be mulitplication operators on M. Mandrekar proved that $R_z R_w^* = R_w^* R_z$ holds if and only if $M = qH^2$ for some inner function q.

A closed subspace N of H^2 is called backward shift invariant if $T_z^* N \subset N$ and $T_w^* N \subset N$. For a backward shift invariant subspace N in $H^2(\Gamma^2)$, two operators S_z and S_w on N are defined by $S_z = P_N T_z P_N$ and $S_w = P_N T_w P_N$, where P_N is the orthogonal projection from $H^2(\Gamma^2)$ onto N. Our theorem is: Let N be a backward shift invariant subspace of H^2 and $N \neq H^2$. Then $S_z S_w^* = S_w^* S_z$ on N holds if and only if N has one of the following forms. (i) $N = H^2 \ominus q_1(z)H^2$, (ii) $N = H^2 \ominus q_2(w)H^2$, (iii) $N = (H^2 \ominus q_1(z)H^2) \cap (H^2 \ominus q_2(w)H^2)$, where $q_1(z)$ and $q_2(w)$ are one variable inner functions.

Also we talk about the case: $S_z^n S_w^* = S_w^* S_z^n$. We give a characterization of backward shift invariant subspaces satisfying $S_z^2 S_w^* = S_w^* S_z^2$. (Received August 23, 2004)