

Meeting: 999, Nashville, Tennessee, SS 14A, Special Session on Graph Theory and Matroid Theory

999-15-31

Guantao Chen* (gchen@gsu.edu), Department of Mathematics and Statistics, Georgia State University, Atlanta, GA 30303, and **George Davis, Frank Hall, Kinnari Patel** and **Michael Steward**. *An Interlacing Result on Normalized Laplacians*.

Given a graph G , the normalized Laplacian associated with the graph G , denoted $\mathcal{L}(G)$, was introduced by Fan Chung Graham and has been intensively studied in the last ten years. For a k -regular graph G , the normalized Laplacian $\mathcal{L}(G)$ and the standard Laplacian matrix $L(G)$ satisfy $L(G) = k\mathcal{L}(G)$, and hence, they have the same eigenvectors and their eigenvalues are directly related. However, for an irregular graph G , $\mathcal{L}(G)$ and $L(G)$ behave quite differently, and the normalized Laplacian seems to be more natural. Cauchy interlacing type properties of the normalized Laplacian are investigated and the following result is established. Let G be a graph and let $H = G - e$, where e is an edge of G . Let $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n = 0$ be the eigenvalues of $\mathcal{L}(G)$ and let $\theta_1 \geq \theta_2 \geq \dots \geq \theta_n$ be the eigenvalues of $\mathcal{L}(H)$. Then, $\lambda_{k-1} \geq \theta_k \geq \lambda_{k+1}$ for each $k = 1, 2, 3, \dots, n$, where $\lambda_0 = 2$ and $\lambda_{n+1} = 0$. Applications are given for eigenvalues of graphs obtained from special graphs by adding or deleting a few edges. (Received July 13, 2004)